UCRL 7224 Rev. II

University of California

Ernest O. Lawrence Radiation Laboratory

THE PHENOMENOLOGY OF THE MASS MOTION OF A HIGH ALTITUDE NUCLEAR EXPLOSION

Facsimile Price $ 3.60
Microfilm Price $ 1.22

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

Livermore, California

DISCLAIMER

UCRL-7224 Rev. II

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Livermore, California

Contract No. W-7405-eng-48

THE PHENOMENOLOGY OF THE MASS MOTION OF A HIGH ALTITUDE NUCLEAR EXPLOSION

Stirling A. Colgate

May 13, 1963

THIS PAGE

WAS INTENTIONALLY

LEFT BLANK

The Phenomenology of the Mass Motion of a High Altitude Nuclear Explosion

Stirling A. Colgate

Lawrence Radiation Laboratory, University of California
Livermore, California

May 13, 1963

ABSTRACT

The persistence of the free expansion of the detonation products of a high altitude detonation is determined by the sum of the stresses of the environment. Below 200 km the classical dynamic friction with the air couples the detonation products to a very much larger mass of air. Above this altitude more uncertain plasma phenomena of magnetic and electrostatic shocks must be considered to determine this coupling. If a cubic scale height of air is raised to a temperature such that its sound speed is greater than escape velocity, it jets upward, and the ionized fraction is stopped and mixed unstably with the earth's magnetic field. The subsequent expansion along the field lines deposits the major fraction of the debris at the opposite conjugate point. The β decay in transit on the various L surfaces reached by unstable mixing determines the high energy electron injection into trapped orbits. For the Starfish event (1.4 megatons at 400 km) theory predicts that the detonation products and a cubic scale height of air expands upwards until stopped by the earth's field at 1000 km. In addition one expects the deposition of the debris in the atmosphere at the southern conjugate point at 250 km altitude dispersed over 300 km due to magnetic field mixing. Approximately 5% β injection into trapped orbits of $L > 1.3$ is expected.

The Phenomenology of the Mass Motion of a High Altitude Nuclear Explosion*

Stirling A. Colgate

Lawrence Radiation Laboratory, University of California

Livermore, California

May 13, 1963

The initial mass-motion of a nuclear explosion is, of course, governed by the hydrodynamics of the free expansion of the detonation products; however, the subsequent persistence of this motion is determined by the sum of the stresses of the environment. At a high enough altitude the expanding detonation products may encounter less than an equivalent mass of air and a free expansion will take place until some additional stress, as, for instance, magnetic field, becomes dominant.

ATMOSPHERE DOMINATED EXPANSION

The pertinent question that separates the behavior of the free expansion from that governed by environmental density is: at what altitude will the integral mass of air above the explosion that 'collides' with the detonation products be greater than the mass of the products ? Since the atmospheric density near the earth varies exponentially with height, the supersonic momentum-conserving expansion for a snowplow-type shock has a velocity distribution

$$u(\theta) = \frac{u_0 M}{M + \int_0^R 4\pi\rho(\theta) r^2 \, dr} \tag{1}$$

*Work done under the auspices of the U. S. Atomic Energy Commission.

where

R = radius of expansion

M = mass of detonation products

u_0 = initial mean expansion velocity

θ = angle with respect to the vertical

and

$$\rho(\theta) = \rho_0 e^{-(r\cos\theta)/h} \tag{2}$$

where

h = scale height of the atmosphere

ρ_0 = atmospheric density (in g/cc) at the detonation point.

The shape of this shock front at some later time t is

$$R(\theta) = \int_0^t u(\theta)\, dt. \tag{3}$$

If we look at this shape after a time long enough so that $R(\theta = 0) \gg h$ and concern ourselves with cases where the total grams per cm^2 of air is large compared to the detonation product mass at a radius large compared to h, namely,

$$\int_0^{R\to\infty} 4\pi r^2 \rho_0\, e^{-r/h}\, dr \gg M, \tag{4}$$

then the radial distribution becomes, Fig. 1

$$R(\theta, t) - u_0 t \frac{M\cos\theta}{8\pi h^3 \rho_0} \text{ for } 1 - \cos\theta < \frac{h}{R(0)}. \tag{5}$$

This distribution is based upon the assumption that the detonation products 'sweep up' the air ahead in a 'snowplow' fashion, i.e., a strong shock with the adiabatic $\gamma \to 1$. If, on the other hand, the detonation products could interpenetrate the ambient air with negligible interaction, then very obviously the free expansion approximation would apply.

DETONATION PRODUCT-AIR INTERACTION

Three interaction mechanisms between the detonation products and the air will be considered: (1) the Coulomb collision dynamic friction (both ion-ion and ion-electron), (2) magnetic shocks, and (3) electrostatic shocks.

Each of these interactions depends upon the initial fractional ionization of the air. Two processes for ionizing the air are evident: first, the pulse of X rays will give an initial partial ionization, and second, each of the above mechanisms of interaction will lead to electron heating and subsequent ionization. The energy required for complete ionization represents a small fractional coupling for any of the above three mechanisms since the energy to be exchanged is very large compared to ionization potentials. In those cases where the initial condition for strong interaction requires a modest fractional ionization the X-ray pulse will satisfy this condition.

From Eq. (5), the effective path in which interaction must take place is approximately h. The flux of 1-kev X rays from a nominal 1 megaton of 1-kev X rays at a distance h, assuming small attenuation, is

$$\phi = W_B/W_x 4\pi(h)^2 = 8 \times 10^{16} \text{ quanta/cm}^2 \qquad (6)$$

where

W_B = 1 megaton

W_x = 1 kev

h = scale height = 50 km.

For a mean absorption coefficient of 1-kev X rays in nitrogen and oxygen of $\sigma = 5 \times 10^{-20}$ cm^2 the available energy per atom becomes

$$W_a = \phi \sigma W_x = 4 \text{ ev.}$$

The energy invested per ion pair is usually 32 ev, and considering that the recombination time at the altitudes in question is longer than the delay for the arrival of the detonation products $\Delta t \approx h/u_0 \approx 0.015$ sec, then a fractional ionization of 12% or greater should exist as a minimum initiation level for any of the strong interaction mechanisms.

COLLISIONAL DYNAMIC FRICTION

The dynamical friction of a test ion slowing down in a plasma has been treated in detail by Kranzer [Kranzer, 1961], who confirms the slowing down approximation of Spitzer [Spitzer, 1962]. The dynamic friction on a test ion can be separated into two parts — that due to collisions with ions and that due to collisions with electrons. The electron temperature is a controlling factor for the electron dynamic friction, because it governs both the thermalization time and the charge state of the ions. (The usual pickup and loss dynamic friction for heavy ions passing through neutral matter is smaller than the free electron friction for electron temperatures > 5 ev.)

The critical region for determining interaction is that region a scale height from the detonation where the products arrive essentially noninteracted with a velocity u_0 and spread in time h/u_0 seconds. The maximum heating rate then becomes

$$\frac{dW}{dt} = \frac{2N_Z \frac{m_e}{M_Z} M_Z u_0^2/2}{m_e^{1/2}(kT_e)^{3/2}/\pi e^4 n_e Z^2 \ell n A} \text{ erg/cc} \cdot \text{sec.} \tag{7}$$

where N_Z in the number density of the detonation products, M_Z its atomic mass, Z its charge, and n_e the plasma electron density.

The loss rate of energy from the plasma will be entirely by line emission (bremsstrahlung being negligible). An approximation to this loss rate can be made according to Post [Post, 1961] and an independent evaluation of the Model C Stellarator impurity radiation loss rate [Hoffman et al., 1962] can be made by assuming that each atom which is not stripped below three electrons has an allowed level 10 ev above the ground state with an excitation cross section $\sigma = 10^{-16}\ \text{cm}^2$. This gives rise to a loss rate

$$dW/dt = n_0 n_e \sigma (kT/2m)^{1/2} h\nu \tag{8}$$

$$= n_0^2\ Z(kT/2m)^{1/2}\ 10^{-15}\ \text{ev/cc}\cdot\text{sec.}$$

An approximation to the ionization state Z can again be made from the work of Post as

$$Z \approx T^{1/2} \quad 5 \leq T \leq 50\ \text{ev} \tag{9}$$

and

$$n_e = Zn_0 \approx n_0 T^{1/2}.$$

Letting $A_Z = 50$ and

$$N_Z = 10^6 \times 6 \times 10^{23}/A_Z \times 4\pi h^3 = 8 \times 10^6/\text{cc};$$

then, equating heating and loss rates, one obtains

$$\frac{2N_Z\left(M_Z u_0^2/2\frac{m}{M_Z}\right)}{m^{1/2}(kT_e)^{3/2}/\pi e^4 n_0 T_e^{3/2}\ \ell n\Lambda} = n_0^2 T_e^{1/2}\left(\frac{kT}{2m}\right)^{1/2} \times 10^{-15}$$

or

$$T = 6 \times 10^{11}/n_0\ \text{ev, for}\ 1.2 \times 10^{10} \leq n_0 \leq 1.2 \times 10^{11}. \tag{10}$$

For densities greater than 1.2×10^{11}/cc the approximations (8) for the resonant radiation loss rate and (9) for the effective Z of the bomb material break down. Since the resonant-state energy is 10 ev, temperatures below 5 ev give rise to an exponentially reduced radiated power, effectively establishing a lower limit to the electron temperature of ~5 ev. In addition, the pickup and loss cross sections of the detonation products traversing neutral matter at 3×10^8 cm/sec establish a minimum effective Z of approximately 2.

At the critical air density, below which there is no interaction, the dynamic friction must be sufficiently large to slow down the detonation products ions in a time $t = h/u_0$. Therefore

$$h/u_0 = \frac{M_Z}{m_e} \frac{m_e^{1/2} (kT_e)^{3/2}}{\pi e^4 n_o Z^2 \ell n \Lambda} \text{ sec.} \tag{11}$$

Using Eq. (9) for n_e and Z^2, and $u_0 = 3 \times 10^8$ cm/sec and $h = 5 \times 10^6$ cm, then

$$1.6 \times 10^{-2} = \frac{M_2}{m_e} \frac{m_e^{1/2} (k)^{3/2}}{\pi e^4 n_0 \ell n \Lambda} \quad \text{where } \ell n \Lambda = 15.$$

Then

$$n_0 = 1 \times 10^{11}/\text{cc}.$$

The nuclear Coulomb scattering criterion for detonation producto air interaction requires a still higher density:

$$n_0 h \sigma_c = \frac{M_Z}{M_{air}} = 3, \quad \sigma_c = \frac{6 \times 10^{-19} Z_1^2 Z_2^2}{\left(M_{air} u_0^2/2\right)} \text{ cm}^2. \tag{12}$$

For $u_0 = 3 \times 10^8$ cm/sec, $\sigma_c = 6 \times 10^{-19}$ cm^2

$$Z_1^2 \approx 50, \; Z_2^2 = 600$$

$$\therefore n_0 = 1.6 \times 10^{13}/\text{cc}.$$

Thus the electron dynamic friction is considerably larger than the nuclear Coulomb scattering.

MAGNETIC SHOCK INTERACTION

The strong interaction by means of a magnetic shock wave depends upon the momentum stress of the detonation products compressing and moving the ambient ionized air. If we first neglect the presence of the ionized air and ask how the detonation products interact with the field alone, then the simple concept of the ionized detonation products blowing a diamagnetic hole in the magnetic field is valid. In general, the hole will expand until the work done against the field equals the kinetic energy of the detonation products. That is,

$$4/3\ \pi R^3\ B^2/8\pi = 4 \times 10^{22}\ \text{erg}$$

$$R = 10^8\ \text{cm for } B = 0.3\ \text{gauss.} \tag{13}$$

The thickness of the boundary X between the diamagnetic sphere of expanding ionized detonation products and the magnetic field is described by the 'M theory' of Rosenbluth [Rosenbluth, 1957]:

$$X = (mc^2/8\pi\ ne^2)^{1/2} \tag{14}$$

where m = electron mass, and

n = number density of electrons associated with detonation products at the boundary.

Since the field can move at the velocity of light, and the detonation products move very much more slowly, there will be a low density front where the momentum stress equals the field pressure. This is the condition for which the above thickness is derived. The physical description of such a layer is that the ions penetrate a distance X into the field with negligible interaction, but the electrons associated with the ions cannot penetrate the field as far because of their smaller Larmor radius. The resulting charge-separation electric field decelerates the ions and causes a transverse drift of the electrons. This electron drift gives rise to a current which is just the diamagnetic current bounding the field from the diamagnetic hole.

For the case of expanding detonation products at a distance h spread out in time h/u_0, the maximum magnetic field interaction distance X becomes;

$$X = 2500 \text{ cm, where } n M_Z u_0^2/2 = B^2/8\pi$$

$$n = 10^4 \text{ for } B = 0.5 \text{ gauss.}$$

This then becomes the maximum length that the detonation products can penetrate the plasma unless an instability takes place which causes a breakdown of the layer. There are two types of instability that can allow the plasma to penetrate the field: one is the hydromagnetic Taylor instability, which depends upon the deceleration of heavy fluid by a light fluid, and the second is an electron velocity phase-space instability in which the electron drift stream is unstable to the generation of plasma oscillations.

The hydromagnetic instability obviously takes place whenever the detonation products density is greater than the field-plus-residual-plasma density. As has been shown experimentally, [Dickinson et al., 1962], one would expect this instability for the conditions of free expansion whenever the field stress decelerates the detonation products. For the case of the earth's field this occurs at R = 1000 km and so does not affect the initial question of coupling to the air. (It will of course, affect the final plasma behavior that we will discuss later.) The case of coupling to a larger mass of air should therefore be Taylor stable.

The sheath instability [Buneman, to be published; Hartman and Sloan, to be published; Shkarofsky, to be published] takes place when the electron Larmor radius becomes equal to the Debye length by the generation of plasma oscillations which allow the plasma to interpenetrate the field. However, this interpenetration can take place only at a rate given as a maximum by Bohm diffusion $D_B = ckT_e/eB$. Since this maximum diffusion corresponds to a random walk step of a Larmor radius in a Larmor period (ions and electrons), then the effective diffusion velocity must always be small compared to the directed stream velocity. Consequently, regardless of the instability a strong momentum interaction between the driving detonation products and the driven air plasma will have taken place. The instability merely affects a major unsettled problem concerning the shock structure [Gardner et al., 1958; Adlam and Allan, 1958; Colgate, 1959; Auer, Hurwitz, and Kolb, 1961; Morawetz, 1962] namely, the immediate division of internal energy among electrons, ions, and ion oscillations. For complete coupling, however, the strength of this interaction requires that either the stress is spread out by instability over a distance large compared to X, or that the ambient magnetic field is compressed to a value so that its pressure equals the time rate of change of the detonation products momentum. That is,

$$W_t/8\pi h^3 = B_{max}^2/8\pi \qquad (15)$$

$$B_{max} = 20 \text{ gauss}.$$

This is not too large a compression, but a shock structure of such a high Alfvén Mach number has not been derived on a self-consistent basis.

NEUTRAL STRESS MAGNETIC SHOCK

A more conservative picture of the magnetic shock interaction can be formed based upon a time-dependent ionization model similar to that formulated by Gerry, Kantrowitz, and Petschek [Gerry, Kantrowitz, and Petschek, 1963]. In this model the time-dependent ionization of the neutral fraction occurring within the moving combined shocked plasma and magnetic field results in the primary momentum stress (Fig. 2). The effective shock thickness becomes the ionization mean free path, which may be longer than the instability wavelengths discussed previously. As a consequence the shorter wavelength instabilities can be neglected, and the ionization mean free path in the shocked plasma becomes the critical parameter that determines coupling. The ionization of the neutral air occurs by:

1. The X-ray pulse (Eq. (6) — the electrons from which may or may not have reached equilibrium by the time of arrival of the shock);
2. The electron ionization;
3. The charged-detonation-products ions and shocked-air ions.

Even assuming equilibrium following the X-ray pulse, this fractional ionization is small compared to unity and so the primary coupling must be determined by 2 and 3. For electron temperatures above 5 ev, the ionization rate of air is approximately constant [Mott and Massey, 1949] ($\overline{\sigma_1 v} \approx 3 \times 10^{-8}$ cm^3/sec), and so electron ionization should be independent of shock velocity for air densities where magnetic shocks are important. The ionization by partially stripped ions, on the other hand, is sensitively dependent upon velocity. Choosing the initial detonation product velocity as the critical velocity for coupling, then the equilibrium charge state must be determined for the ionization rate. Knipp and Teller [Knipp and Teller, 1941] have estimated the effective charge of a high-Z atom slowing down in a neutral gas and have shown that capture and loss are in equilibrium when $u_0 = v_e/1.5$, where v_e is the bound orbital electron velocity. Choosing $u_0 = 3 \times 10^8$ cm/sec, then the ionization energy corresponding to $v_e = 4.5 \times 10^8$ cm/sec is 80 ev or $Z_{eff} \approx 3.5$.

The cross section for nitrogen ionization by an ion of charge Z_{eff} and velocity u_0 is

$$\sigma \approx 5 \times 10^{-16} (Z_{eff})^2 = 7 \times 10^{-15} \text{ cm}^2.$$

The detonation product density at a distance h is

$N_D = M \times 6 \times 10^{23} / \overline{A}\, 4/3 \pi h^3$. For $M = 10^6$ gram and $\overline{A} = 20$,

$$N_D = 3 \times 10^7/\text{cc}. \quad (16)$$

Therefore the mean free path λ of a neutral air atom in the flux of detonation products becomes

$$\lambda = 1/N\sigma = 5 \times 10^6 \text{ cm}.$$

The electron density such that electron ionization will have contributed equally to the ionization of the air within the time of the passage of the detonation products is:

$$n_e = u_0 / \overline{\sigma_i v}\, \lambda = 2 \times 10^9 \text{ electrons/cc}. \quad (17)$$

In summary, a strong interaction between detonation products and the combined air and magnetic field is predicted for an ionization-type hydromagnetic shock where the shock thickness due to either ions or electron ionization is large and comparable to the scale height of the atmosphere. During the expansion phase such a large shock thickness should be stable until the magnetic field stress reverses the expansion.

Since the ionization magnetic shock by debris ions becomes the dominant shock mechanism above 350 km, it is pertinent to demonstrate that the high debris charge state of 3.5 may have had a chance to reach and maintain equilibrium before having expanded to a scale height in radius. It is evident that a free expansion in a vacuum will permit the detonation products to cool by adiabatic expansion and radiation leading to recombination and a lowering of charge state. However, because of collisions with air molecules the detonation products will be heated, but this heating is small compared to blackbody radiation (even at 1 ev). Consequently, it is only after the detonation products have expanded to the point where they are optically thin, yet an air-atom collision range thick, that the temperature will rise and the charge state become high. Naturally, if the detonation products are a collision range thick, there is more than ample time for charge equilibrium to be reached.

The range from Eq. (11) becomes

$$N_D R = 5 \times 10^{17} \text{ atoms/cm}^2$$

where

N_D = detonation product atom density

R = radius of detonation products.

Solving for R analogously to Eq. (16) gives

$$R = 1 \text{ km}, \; N_D = 5 \times 10^{12}/\text{cc}.$$

Using Eq. (8) for the resonant line emission power loss (an upper limit because of the available states and self-absorption) and assuming $n_c = 3.5\ n_0$ gives a power loss rate

$$p_{rad} = 10^{12} \text{ ergs/cm}^2 \text{ sec}.$$

The air density ρ_a such that the collisional heating equals the power radiated

$$p_{rad} = \rho_a u_0^3/2$$

becomes

$$\rho_a = 6 \times 10^{-14} \text{ g/cm}^3.$$

At lower air densities the balance will occur at a larger detonation product radius since the heating rate decreases as $1/N_D$ for R > air atom range; whereas the radiation decreases as $1/N_D^2$. Consequently, over the air density range of interest ($10^{-11} \geq \rho_a \geq 10^{-15}$) the detonation-product charge state should be maintained at an equilibrium value of $Z \approx 3.5$ until slowed down.

ELECTROSTATIC SHOCK INTERACTION

The electrostatic shock depends upon the growth of large-amplitude plasma oscillations and/or ion sound waves due to the two-stream interaction. Buneman [Buneman, 1959] has shown that the electron-ion two-stream instability grows to saturation in roughly 30 plasma periods. Saturation occurs when the electron stream kinetic energy is randomized by nonlinear effects to a quasi-thermal distribution. When the resulting electron thermal velocity (v_{eth}) equals the relative stream velocity, linear-unstable growth stops. The ion-wave instability as shown in detail by Stringer [Stringer, 1961] grows provided the ion temperature is small compared to the electron temperature and provided the electron thermal energy is less than the ion stream energy. On the other hand, the maximum ion wave amplitude (measured in energy) is limited in linear theory to the electron temperature. The possibility of further nonlinear instability growth has been investigated numerically by Hartman and Colgate [Hartman and Colgate, 1962], and although large amplitude growth has been confirmed for electron ion mass ratios up to 100, some doubt exists for mass ratios as great as 10^5 for the present case.

Accepting the more conservative result of the saturation of the electron instability at $v_{eth} = u_0$, this gives a two-stream dynamic friction that will maintain the electron temperatures at $m\,u_0^2/2 \approx 50$ ev. This magnitude of the friction is then limited by the plasma radiation loss rate at 50 ev. Equating this loss rate to the detonation products energy input rate determines a minimum density for interaction. From Eqs. (8) and (9),

$$n_0^2\, T^{1/2}\, (kT/2m)^{1/2}\, \sigma(h\nu)k = W u_0/8\pi h^4 \qquad (18)$$

where

$(h\nu) = 10$ ev, $(kT/2m)^{1/2} = u_0 = 3 \times 10^8$ cm/sec, $T = 50$ ev, and

$\sigma = 10^{-16}$ cm^2.

Then

$$n_0 = 1.4 \times 10^{10}/\text{cc}.$$

This is a stronger interaction than the electron Coulomb dynamic friction, but certainly weaker than the magnetic shock.

ENERGY CONSERVING INTERACTION AND EXPANSION

The magnetic shock ensures strong interaction essentially independent of density, but the limit of free expansion behavior is determined by the radial distribution of Eq. (5). Choosing $M = 10^6$ grams, and $h = 50$ km, then ρ_{min} becomes:

$$\rho_{min} = M/8\pi h^3 = 3 \times 10^{-16} \text{ g/cc} \tag{19}$$

or $n_0 = 1.25 \times 10^7$/cc.

Above this altitude free expansion takes place. Below this altitude the combination of detonation products and heated air will expand upward at a velocity given by Eq. (1) provided the resonant-line radiation cooling of the air behind the shock justifies the approximation of a snowplow shock, that is, that the internal energy is radiated away as fast as it forms. However, for densities less than the limits of Eq. (10), that is, $10^{10} < n_0 < 10^7$/cc, the resonant radiation loss very rapidly becomes negligible, because air atoms are stripped of electrons below the lithium 3-electron configuration, and only levels of large energy gap and small cross section remain. The radiation power loss rate from air decreases by two orders of magnitude as the temperature increases from 20 to 100 ev [Post, 1961]. Consequently, in this intermediate density region the subsequent expansion of the high-temperature air plasma controls the hydrodynamics more than the directed momentum of the detonation products expansion.

In order for this 'bubble' of high temperature plasma to expand upwards through the remaining atmosphere, it must comprise a volume of roughly a cubic scale height, otherwise the mass/cm^2 above will be greater than that in the 'bubble', thereby limiting upward motion. The expansion velocity upward is then sound speed within the bubble. Assuming a specific heat per electron of 3 to include the heat of ionization and the radiation during ionization and a volume from Eq. (5), the expansion velocity u_{ex} becomes

$$24\pi\rho\, h^3\, (u_{ex})^2/2 = W_t$$

or

$$u_{ex} = \left(W_t/12\pi\rho\, h^3\right)^{1/2}\ \text{cm/sec.} \tag{20}$$

The bubble will cool by both radiation and expansion, but in general once the temperature has run away above the resonant line-emission range, the adiabatic cooling will dominate. As pointed out earlier, the subsequent expansion will be limited by the stress of the magnetic field at roughly 1000 km for a megaton. Assuming a $\gamma = 5/3$ adiabat, the final temperature T_f becomes

$$3\, n_e\, k\, T_f\, h^3 = W_t\, (h/R)^{3(\gamma-1)} \tag{21}$$

where $n_e = 8.2\, n_0$ for $T_i > 50$ ev.

Then

$$T_f = W_t/25\, n_0\, k\, R^2\, h. \tag{22}$$

For $R = 10^3$ km, $h = 50$ km, $W_t = 1$ megaton, and

$$T_e = 1.8 \times 10^{10}/n_0\ \text{ev.}$$

When the bubble material is stopped by the magnetic field, one would expect some heating of the electrons from the inelastic energy of the ions. The fractional cooling to be expected within the time of interaction between the bubble material and magnetic field (assuming T_e lies within the resonance radiation range) by adiabatic cooling is by Eq. (8):

$$f = \frac{(h/R)^3 \, n_e \, \sigma(2KT/m)^{1/2} \, (h\nu)k \, R/u_{ex}}{M_{air} \, u_{ex}^2 /2}$$

$$= n_0 \, T/u_{ex}^3 \, 1.6 \times 10^8. \qquad (23)$$

This fraction is small ($\sim 10^{-2}$) for the largest n_0 ($\sim 10^{10}$/cc), and so the radiative cooling of the plasma after expansion is negligible. So also is the heating for $T \geq T_{max}$. T_{max} is that electron temperature such that the electrons can make one thermal exchange time with themselves within the expansion time.

$$1/n_e \, \sigma v = R/u_{ex}$$

$$n_e = n_0 \, T^{1/2} \, (h/R), \quad \sigma = 6 \times 10^{-13}/T^2 \ \text{cm}^2 \qquad (24)$$

$$v = 5.5 \times 10^7 \, T^{1/2} \ \text{cm/sec}$$

or

$$T = 0.3 \, n_0/u_{ex} \ \text{ev}.$$

When the adiabatic temperature of Eq. (22) exceeds the heating temperature given by Eq. (24), then no further change should be expected. At $n_0 = 10^{10}$/cc the electron temperature should stabilize at roughly 10 ev from both effects.

MAGNETIC FIELD LIMITED EXPANSION AND FINAL DEPOSITION

When the plasma of air and detonation products is stopped by the stress of the magnetic field at a distance R, we expect a Rayleigh-Taylor unstable mixing between the two because the density of the incident plasma is greater than the ambient density (Fig. 3). The linear growth of a perturbation of amplitude A_0 is [Dickinson et al., 1962]:

$$A = A_0 \exp [g/2\pi\lambda)^{1/2} t] \tag{25}$$

where g is the acceleration and λ the wavelength. For a linear deceleration taking place in a distance R, the growth becomes:

$$A = A_0 \exp [R/\pi\lambda]^{1/2}. \tag{26}$$

Only the nonlinear phase of instability growth (where $A \geq \lambda/2$) will irreversibly mix the plasma with the field. Consequently, if we wish to know the irreversible mixing of the plasma onto magnetic flux surfaces other than the one defined by the explosion, we must ask what is the largest wavelength that requires the fewest generations of growth to reach an amplitude greater than $\lambda/2$. Then, because of the faster growth rate of smaller wavelengths, all smaller wavelengths should have reached the nonlinear limit too.

For a rising bubble or jet of plasma of form approximated by Eq. (5) the wavelength $\lambda = R/\pi$ has an amplitude of approximately $\lambda/2$ so that during deceleration by the field by Eq. (26) one generation further growth can take place within the nonlinear limit. Therefore, the plasma should penetrate the field with a mixing length R/π.

The subsequent motion of the plasma will be an expansion along the lines of force, since in the parallel direction less stress is exerted by the field. The principle direction of this expansion will be away from the hemisphere of detonation because of the following stresses, all of which tend to direct the plasma from a northern hemisphere explosion towards a southern hemisphere deposition:

1. The verticle motion of initial expansion Eq. (5) has a component of momentum along the field lines towards the far conjugate point.
2. The ambient field initially diverges in a direction towards the far conjugate point.
3. The distorted field diverges more strongly because of the 'line tying' in the partially ionized atmosphere at the near conjugate point.

For these reasons the plasma will travel along the lines of force until they intersect the earth's atmosphere at the far conjugate point with a distribution determined by the magnetic flux surfaces intersection with the earth. Since we have already shown that due to the low density of the plasma at the time of mixing the interaction should be energy-conserving, the velocity along the lines will be sound speed, namely, u_{ex}.

DEPOSITION IN THE ATMOSPHERE AT THE CONJUGATE POINT

On intersection with the atmosphere at the conjugate point, the plasma will have a width (longitude) as well as latitude determined by both the mixing length R/π and the diamagnetic pressure. Since the plasma expands into a tube roughly 6000 km long, the radius for a pressure the same as the bubble of R = 1000 km is also approximately 300 km. The altitude of deposition will be determined by the electron dynamic friction and the nuclear Coulomb scattering. The impact velocity u_{ex} determines which of these two dynamic frictions will dominate. If the atmosphere is not heated by impact to a temperature high enough so that the free electron density is comparable to the nuclear density, then the electron dynamic friction becomes small for an impact velocity such that the relative kinetic energy of an electron is less than the binding energy in the atom, namely:

$$m\,u_{ex}^2/2 \geq 7 \text{ ev}$$

or

$$u_{ex} \geq 1.5 \times 10^8 \text{ cm/sec.} \tag{27}$$

Below this velocity the bound-free electron dynamic friction becomes small and so the heating of the atmosphere is small until the nuclear scattering becomes large. By then the incident plasma is stopped and the question of heating the atmosphere to ionization becomes unimportant. Only in the velocity range $u_{ex} \approx 1.5 \times 10^8$ does the question of heating become important. The energy available is reduced to $W_t/3$ due to radiation loss during ionization and recombination, and this energy is deposited over an area $(R/3)^2$. Assuming twice the ionization potential in electron volts is required to ionize each atom because of radiation loss, the maximum density N_{max} that could be ionized is:

$$h\,N_{max}\,2\,E_i\,k = W_t/3(R/3)^2 \tag{28}$$

for

$$E_i = 7 \text{ ev.}$$

$$N_{max} = 1.2 \times 10^9/\text{cc.}$$

This is too low a density to stop the plasma by electron free-free dynamic friction by Eq. (11), so the dominant stopping will be by bound-free for $u_{ex} > 1.5 \times 10^8$ cm/sec and by nuclear-Coulomb for $u_{ex} < 1.5 \times 10^8$. The nuclear-Coulomb dynamic friction is given by Eq. (12) on the basis that both nuclei of a collision are unscreened by bound electrons. This is the case if the impact parameter $(\sigma/\pi)^{1/2}$ is small compared to the orbit of the k-shell electron.

The minimum incident energy for which Eq. (12) is valid is determined by

$$\frac{4\pi e^4}{(Ry P^2 Z^2 k^2)} = \frac{4\pi e^4 Z^4 \ell n \Lambda}{k^2 (M_{air} u_{ex}^2 /2)^2} \tag{29}$$

where Ry = Rydberg constant = 13 ev

and $\ell n \Lambda = 10$

giving

$$M u_{exp}^2 /2 = 13 \text{ kev}.$$

At lower incident energies the term that sums the multiple scattering ($\ell n \Lambda$) approaches unity and the cross section approaches the classical Rutherford cross section at $M u_{ex}^2 /2 = 4$ kev. Therefore an approximately constant stopping cross section of $\sigma = 1.5 \times 10^{-17}$ cm^2 can be given for the energy range $4 \leq Mu^2/2 \leq 13$ kev. This corresponds to deposition at an altitude of 250 km.

Since for the very much higher energy region the bound-free dynamic friction should stop the incident plasma at the same altitude (Eq. (11); $u_{exp} = u_0/2$), we can feel reasonably confident of a deposition height of 250 km. The distribution according to magnetic flux surface should therefore be compared to the earth's field at this altitude.

MAGNETIC FIELD GRADIENT DRIFT

In addition to the spreading in flux surface due to the Taylor instability mixing, there will also be a drift in longitude due to finite Larmor orbit drift in the gradient of the magnetic field. This depends upon the following parameters and assumptions:

1. The region of most effective field gradient may be the small region of high gradient in the initial field distortion or the dipole gradient effective during the transit to the conjugate point.

2. The transverse electric field established by charge separation may, or may not dominate the ∇B drift.

3. The transverse ion velocity and equilibrium charge state effect both the $\underset{\sim}{E} \times \underset{\sim}{B}$ and $\nabla \underset{\sim}{B}$ drift.

Unfortunately, both regions of field — the distortion loop and dipole gradient — are effective in producing drift with the greatest uncertainty concerning the detailed loop shape and gradients. In addition the relative importance of the electron stress versus ion stress in the formation of the ∇B^2 of the loop is dependent upon a detailed knowledge of the nonlinear instability mixing behavior. If the interaction boundary is a stable 'M' layer as already discussed, then the ion stress is balanced by an electric field from charge separation and only the electrons drift transverse to the magnetic field. On the other hand, if the layer is unstable, the stress will be divided according to the respective temperatures — which in turn are dependent upon the form of instability.

The drift in the dipole field and in the distortion loop will be calculated on the a priori assumptions:

(1) The electron temperature is determined by the linear processes only and will be taken as 10 ev from Eq. (24) for the limited initial density region $10^9 < n_0 < 10^{10}$.

(2) The ion transverse velocity in the earth's field is the expansion velocity u_{ex}.

For these assumptions the drift velocity v_D becomes

$$v_D = u_{ex} r_L \nabla B/B$$

$$r_L = \text{ion Larmor radius} = M u_{ex}/Z_e B. \qquad (30)$$

The charge state Z is determined by the recombination rate in the expanded plasma at the density $n_0 (h/R)^3$ and $T_e \approx 10$ ev. This rate (for 3-electron configurations) is given by Post [Post, 1961] as

$$n\sigma v = n \frac{0.8 \times 10^{-16} Z_{eff}^4}{T_e^{3/2}} k_3(y) \qquad (31)$$

where $k_3(y)$ will be close to unity for $kT_e' << E_i$, the ionization potential of the state Z_{eff}.

For a recombination time of 1 sec,

$$Z_{eff} = 1.7 \times 10^3/n^{1/4}$$

for $T_e = 10$ ev

giving $Z_{eff} \approx 100$.

This means that for the density range considered, radiative recombination is much too slow a process to occur within the available transit time of the plasma. Consequently, the charge state will be determined by the maximum charge stripped state during the initial detonation products-air interaction. This in turn depends upon the runaway electron temperature in this region — and therefore upon the form of the interaction. From Eqs. (9) and (10) we will assume the air has been completely stripped to a mean charge of 7. The subsequent charge exchange with neutral air atoms in traversing a distance of the earth's radius at an ambient density of 10^4/cc is small. From Mott and Massey [Mott and Massey, 1949]

$$\sigma = 10^{-16} \left(u_{ex}/2.5 \times 10^8\right)^2 \text{ where } u_{ex} \leq 2.5 \times 10^8 \text{ cm/sec.} \qquad (32)$$

The fraction charge exchange to Z = 6 is less than

$$f \leq n\sigma \; L = 6 \times 10^{-4}$$

$$(L = \text{earth's radius} = 6 \times 10^{8} \text{ cm})$$

and so is negligible.

The Larmor radius then becomes

$$r_L = 5.3 \times 10^{-4}/B \; u_{ex} = 7 \times 10^{-4} \; u_{ex} \text{ cm in earth's field.}$$

To calculate the drift in the earth's field the dipole field gradient is

$$\nabla B/B = 3/L$$

so that the drift velocity becomes:

$$v_D = u_{ex}^2 \; 3.5 \times 10^{-12} \text{ cm/sec,}$$

and the drift D becomes

$$D = Lv_D/u_{ex} = 2 \times 10^{-3} \; u_{ex} \text{ cm.} \qquad (33)$$

Some fraction of the detonation products as a precursor may not have been subjected to the runaway temperature region of the mean interaction and so may have a much lower state of ionization. Choosing the extreme of charge one for a fission product the drift becomes

$$D = 10^{-1} \; u_{ex} \text{ cm.} \qquad (34)$$

This distance is smaller than the mixing length spread provided $u_{ex} < u_0$ and so should not be observable.

The drift in the formation of the distortion loop may be considerably larger. Again assuming the stress on the field is created by the ions, then the logarithmic gradient of the field should be roughly the ion Larmor radius for a length of time corresponding to the intermixing of plasma and field. Since the drift velocity becomes roughly half the incident velocity u_{ex}, and the period of drift is $R/\pi \, u_{ex}$, the total drift becomes

$$D \approx R/2\pi. \qquad (35)$$

BETA INJECTION FROM RADIOACTIVE DEBRIS

The injection of energetic electrons from the β decay of the radioactive debris should occur during the transit of the debris to the conjugate point. Only the decay after the unstable mixing of the plasma and field should permit injection onto flux surfaces other than the one intersecting the detonation point. Since the time to reach the unstable mixing condition R/u_{ex} is in general shorter (< 1 sec) than the major fraction of the β decay (~ 3 sec), and the flight time to the conjugate point longer than 5 seconds, then most of the decay β's will be injected into flux surfaces above the atmosphere. However, only those flux surfaces that do not intersect the earth's atmosphere for a reasonable mirror ratio will show long β lifetime. This minimum L surface [McIlwain, 1961] is approximately L = 1.3 so that only that fraction of the debris reaching this L surface and above by unstable mixing should contribute to the long-lifetime β injection.

NEUTRAL AND SINGLE-CHARGED DETONATION PRODUCT BEHAVIOR

The considerations so far have been for an average behavior and only the dominant interactions have been considered. It is evident, however, that the charge state will fluctuate and the effects of a statistically small fraction may be important. In particular, the singly-charged high energy tail of the detonation products may drive the magnetic field and partially (X-ray) ionized air away from the remaining neutral fraction before ionization is complete, and secondly, a small fraction of the debris neutralized by charge exchange will expand across the field lines to give β-decay injection at higher L values than would be reached by the Taylor instability mixing.

BETA INJECTION BY NEUTRAL DEBRIS

The fraction of the debris that escapes as neutral atoms should be generated predominantly by the slower moving debris that expands upward after intersecting with the air. The higher velocity precursor debris has a smaller probability for escaping as neutrals. The cross section for pickup from neutral air is [Mott and Massey, 1949]

$$\sigma_{10} = 10^{-15}\left(\frac{10^8}{u_{ex}}\right)^5 \text{ cm}^2 \text{ for } u_{ex} \geq 10^8 \text{ cm/sec} \tag{36}$$

$$= 10^{-15} \text{ cm}^2 \text{ for } u_{ex} \leq 10^8.$$

On the other hand, the air through which the debris is rising and expanding will be ionized by the debris. Only that density region which is not fully ionized by the expanding debris will contribute to the charge neutralization. Assuming the debris temperature during this phase of expansion is greater than 5 ev, the ionization rate will be temperature-independent and the expanded density determines when the further ionization of ambient neutrals is unlikely. This density is determined by

$$n_e \overline{\sigma_i v} = u_{ex}/h, \tag{37}$$

giving

$$n_e = 5\, u_{ex}.$$

The expansion of the plasma at initial density 6 n_0 to the density n_e takes place in a distance r into a solid angle of 1 radian such that:

$$r = h(3 \times 6\, n_0/5\, u_{ex})^{1/3} \tag{38}$$

and the ambient neutral density becomes:

$$N_n = n_0 \exp\left[-(4\, n_0/u_{ex})^{1/3}\right]. \tag{39}$$

The neutral fraction becomes

$$f = N_n \sigma_{10} h \tag{40}$$

$$= 5 \times 10^{-9} n_0 \exp\left[-(4 n_0 / u_{ex})^{1/3}\right] \text{for } u_{ex} < 10^8.$$

This is a major fraction only for $n_0 \approx 10^8$, or an altitude greater than 550 km.

The β injection from this neutralized debris will occur according to the usual $t^{-1.2}$ power law for fission product decay. It should be noted that for fairly low altitude detonations where u_{ex} is very small (Eq. (20)) — of the order 10^6 cm/sec — neutralization will take place by recombination within the expanding bubble. This debris will be rising slowly so that both β decay and sunlight will cause reionization. The decay spectrum will change within this time to give a different injection spectrum as a function of L value. In addition, a significant change in the trapped β spectrum should be observed for flux surfaces reached in times of the order of an hour, and the β velocity distribution function at the equator will depend upon the debris reaching their corresponding geopotential.

AURORAL EFFECTS

The deposition of the detonation products and air plasma at the far conjugate point will heat up the air in which it is deposited, causing the air to expand upward and intersecting some of the trapped β-decay electrons. The optical pattern will correspond to the illumination of the flux surfaces on which high-energy electrons are trapped and hence to the gradient field drift separated charge state of the β-decaying debris.

As previously discussed, the heating takes place at 250 km, depositing W_D electron volts per atom during the transit time of the plasma. This energy is:

$$W_D = W_t/3\ N_t = 4 \times 10^{22}/3\ h(R/3)^2\ nk \quad (41)$$

for $R/3 = 300$ km, $n = 2 \times 10^{10}$ and

$$k = 1.6 \times 10^{-12} \text{ ergs/ev},$$

then $W_D = 40$ ev/atom.

The transit time is so long, however, (up to tens of seconds) that this energy will be almost entirely radiated away as it is deposited. The residual temperature will therefore be determined by the radiation properties of the air — which becomes transparent at roughly 1/2-ev temperature. The vertical expansion velocity corresponding to twice sound speed at this temperature will be approximately 3×10^5 cm/sec, permitting the air to reach 500 to 1000 km altitude in several minutes. The optical radiation corresponds to the excitation of the air with approximately 10% fluorescent efficiency for an energy deposition rate corresponding to the usual minimum ionization rate of 2 Mev/g/cm^2.

The β-decay electron density for uniform phase-space injection into a magnetic tube of force 300 km wide and 6000 km long for an injection time longer than 5 seconds becomes:

$$n_\beta = W_\beta/E_\beta\ \text{Vol} = 2.5 \times 10^3 \text{ electron/cc} \quad (42)$$

where

W_β = β-decay energy = 0.05 W_t,

$\bar{E}_\beta$ = mean β energy = 1 Mev, and

Vol = Volume of flux tube.

The radiated energy loss rate then becomes

$$E_R = c n_\beta \rho \times 2 \times 10^6 k = 10^{-6} \text{ ergs/cc} \cdot \text{sec} \quad (43)$$

where c = electron velocity

ρ = density after 10-fold expansion

= 5×10^{-14} g/cc

= fluorescent efficiency $\approx$ 10%.

The total optical brightness B_r for a transparent thickness of 300 km becomes

$$B_r = 3 \times 10^7 E_r = 30 \text{ erg/cm}^2 \cdot \text{sec}. \tag{44}$$

The heating rate of the air is ten-fold bigger than Eq. (43) and corresponds to 1 ev per atom during the 100 seconds of expansion. This is enough to enhance the expansion so that some air would be expected to ionize and expand back along the lines of force from both conjugate regions to meet at the equator. The energy loss rate per electron is 2000 ev/sec, but during the available time before the gradient field drift carries the electrons eastward the fractional energy loss is small. The energy loss occurs for a time:

$\tau = (R/3)/V_D$ = 30 sec where

$V_D = 10^6$ cm/sec from Eq. (27) for a 1-Mev electron

so that only the low energy electrons with larger energy loss rate and slower drift will be lost in the rising air mass.

REFERENCES

Adlam, J. H. and J. E. Allan, Second International Conference on the Peaceful Uses of Atomic Energy, 31, 221, 1958.

Auer, P. L., H. Hurwitz, and R. W. Kolb, Phys. Fluids, 4, 1105, 1961.

Buneman, O., Phys. Rev., 115, 503, 1959.

Buneman, O., (Electronics Laboratory, Stanford University, to be published).

Colgate, S. A., Phys. Fluids, 2, 485, 1959.

Dickinson, H. et al., Phys. Fluids, 5, 1048, 1962.

Gardner, C. et al., Second International Conference on the Peaceful Uses of Atomic Energy, 31, 324, 1958.

Gerry, E. T., A. R. Kantrowitz, and H. E. Petschek, Bull. Am. Phys. Soc., 2, 158, 1963.

Hartman, C. W. and D. H. Sloan, Turbulence in crossed-field electron streams, University of California Report (to be published in Phys. Fluids).

Hartman, C. W. and S. A. Colgate, Bull. Am. Phys. Soc., 7, 273, 1962.

Hoffman, F. W. et al., Abstract F-7; A. S. Bishop et al., Abstract F-8; and E. Hinnov, Abstract F-9; Am. Phys. Soc. Fourth Annual Meeting, Division of Plasma Physics, Atlantic City, New Jersey, Nov.-Dec., 1962.

Knipp, J. and E. Teller, Phys. Rev., 59, 659, 1941.

Kranzer, H. C., Phys. Fluids, 4, 214, 1961.

McIlwain, C. E., J. Geophys. Res., 66, 3681, 1961.

Morawetz, C. S., Phys. Fluids, 5, 1447, 1962.

Mott, N. F. and H. S. W. Massey, Theory of Atomic Collisions, p. 275, Oxford University-Clarendon Press, London, 1949.

Post, R. F., J. Nucl. Energy, 3, 273, 1961.

REFERENCES (Continued)

Rosenbluth, M. N. in Magnetohydrodynamics (R. Landshoff, ed.) p. 57, Stanford University Press, 1957.

Shkarofsky, J. P., R. C. A. Report 7-801-13 (to be published).

Spitzer, L., Jr., Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York, 1962.

Stringer, T. E., IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Salzburg, 4-9 September, 1961, Paper 53.

/rd

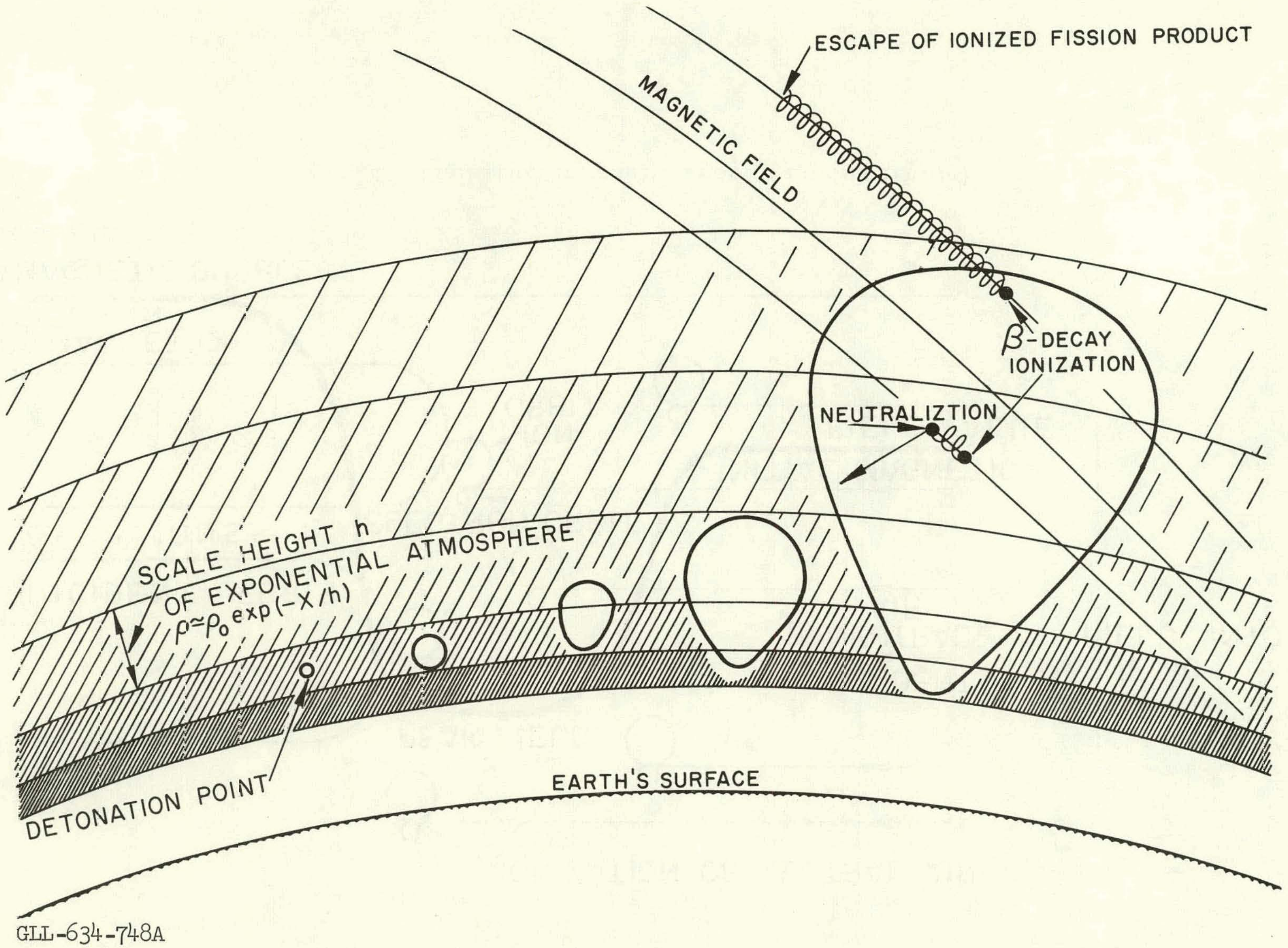

Fig. 1. The time sequence of the expansion of detonation products in a stratified atmosphere is represented by the approximate snowplow shock contour. The air and detonation products in this case are considered neutral and only subsequent fission decay gives debris interacting with magnetic field.

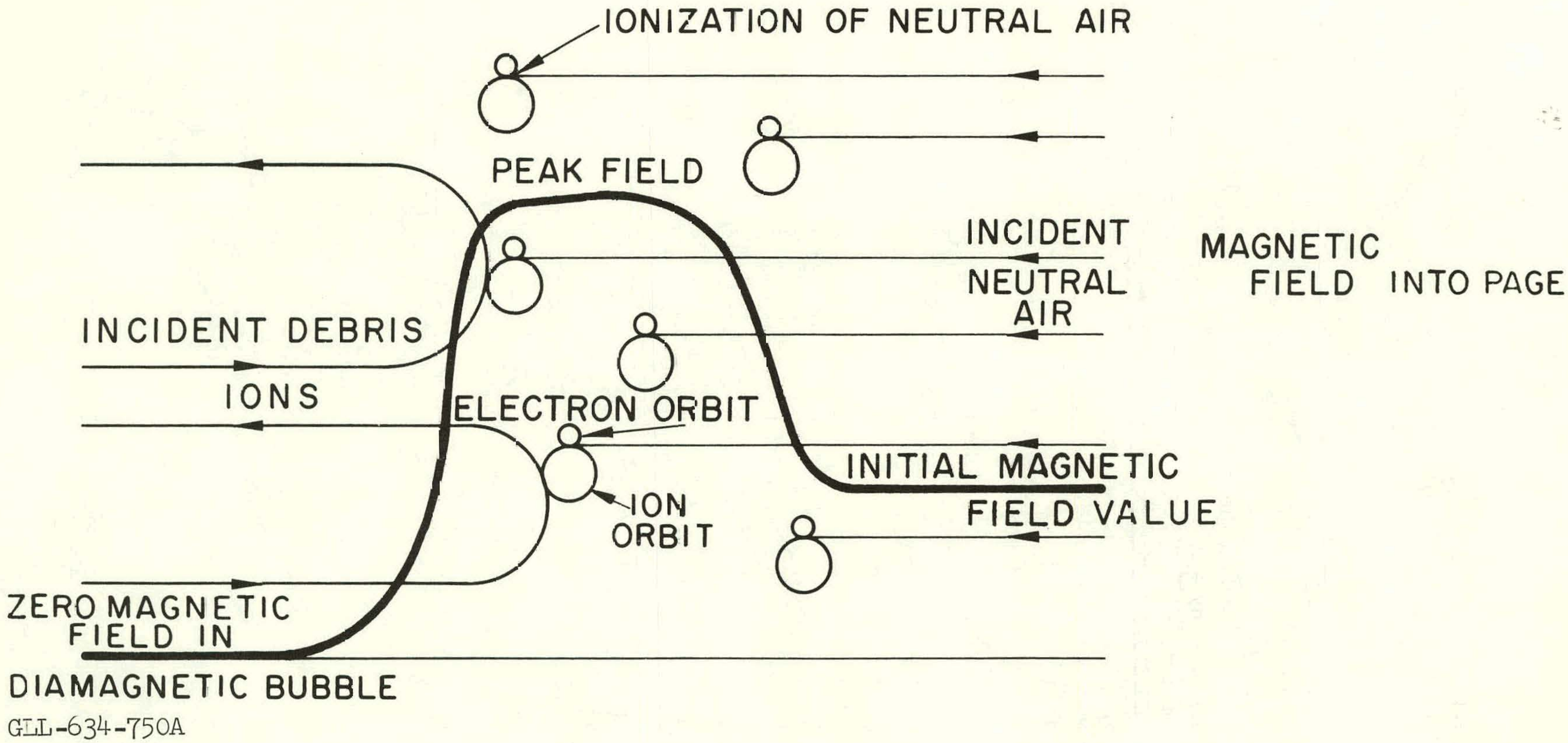

Fig. 2. Ionization magnetic shock (moving frame).